

Computation of the Electric Field

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1 Introduction

Figure 1: An observer (orange dot) at point P near a negative charge ($q_1 = -1nC$).

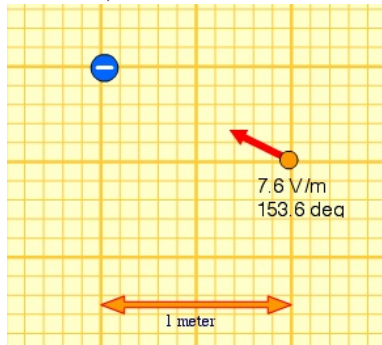
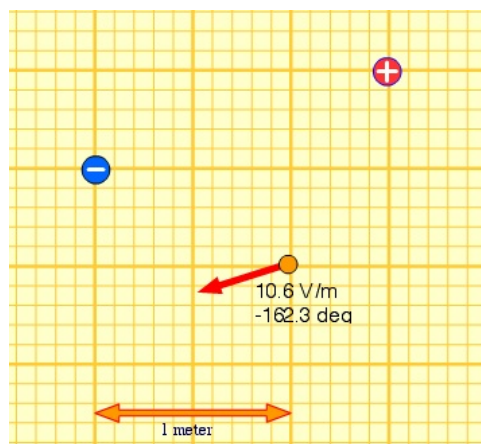


Figure 2: An observer at point P near a negative charge ($q_1 = -1nC$) and a positive charge ($q_2 = +1nC$).



In these notes, we will go through the method of computing the electric field produced by electric charges. In Figure 1, the observer (represented by the orange dot) is standing at point P near a negative charge of $q_1 = -1nC$. As you have seen from lectures, the negative charge will create an electric pointing toward itself, so the observer in Figure 1 will experience an electric field point in the northwest direction. This electric field is represented by the vector (arrow). It is our purpose to show you how the electric field vector is computed.

Figure 2 has two charges q_1 and q_2 , but the calculation is very similar. We will go through the calculation in great details below. We will make use of the following equation frequently:

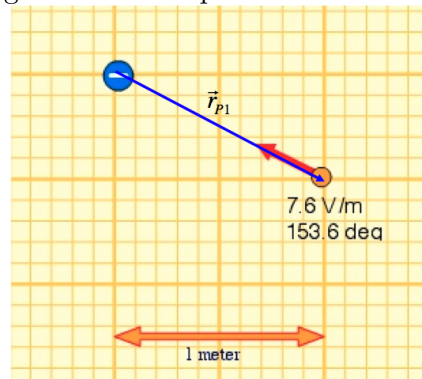
$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \quad (1)$$

2 One Charge

Here we will compute the electric field vector produced by $q_1 = -1nC$ at the position of the observer (the orange dot) at point P .

First we define the displacement vector \vec{r}_{P1} that points from q_1 to P as in Figure 3. Remember that this vector must *always* point from the charge to the observer, *never* the other way around.

Figure 3: The displacement vector \vec{r}_{P1} .



From Figure 3 one can immediately write down the vector and compute its magnitude:

$$\begin{aligned} \vec{r}_{P1} &= 1\hat{i} - 0.5\hat{j} \\ \Rightarrow |\vec{r}_{P1}| &= \sqrt{1^2 + 0.5^2} = 1.12. \end{aligned}$$

where we have omitted the unit of r for simplicity.

Next we want to calculate \hat{r}_{P1} :

$$\hat{r}_{P1} = \frac{\vec{r}_{P1}}{|\vec{r}_{P1}|} = \frac{1\hat{i} - 0.5\hat{j}}{\sqrt{1^2 + 0.5^2}} = 0.89\hat{i} - 0.45\hat{j},$$

which we can now put into equation (1) to get \vec{E}_1 :

$$\begin{aligned}\vec{E}_1 &= \frac{q_1}{4\pi\epsilon_0 r_{P1}^2} \hat{r}_{P1} \\ &= \frac{-1nC}{4\pi\epsilon_0 (1.12)^2} (0.89\hat{i} - 0.45\hat{j}) \\ &= (-6.38\hat{i} + 3.22\hat{j})V/m.\end{aligned}\tag{2}$$

Note that the unit can be given either in V/m or N/C .

The above gives the electric field *vector* \vec{E}_1 , from which we can calculate the magnitude easily:

$$|\vec{E}_1| = \sqrt{6.38^2 + 3.22^2} = 7.15V/m.$$

Our answer of $7.15V/m$ is sufficiently close to the value of $7.6V/m$ given in Figure 1. The discrepancy is caused by the position of the charge being slightly off the grid to the right, as you can see if you look carefully at the figure.

2.1 Two Charges

The only difference between Figure 1 and 2 is that in Figure 2 there is another charge q_2 on the right. The position of q_1 has not changed. If we call the electric field produced by q_1 and q_2 as \vec{E}_1 and \vec{E}_2 respectively, the total electric field is simply given by:

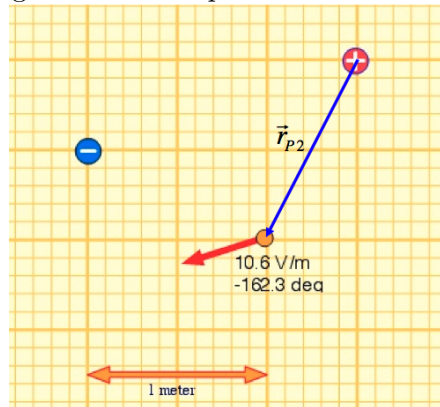
$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2,$$

where \vec{E}_1 was already computed in equation (2). To find \vec{E}_2 , we only need to follow the same steps in section 2.

The vector \vec{r}_{P2} is defined in Figure 4, from which \vec{r}_{P2} , $|\vec{r}_{P2}|$ and \hat{r}_{P2} can be computed:

$$\begin{aligned}\vec{r}_{P2} &= -0.5\hat{i} - 1\hat{j} \\ |\vec{r}_{P2}| &= \sqrt{0.5^2 + 1^2} = 1.12 \\ \hat{r}_{P2} &= \frac{\vec{r}_{P2}}{|\vec{r}_{P2}|} = \frac{-0.5\hat{i} - 1\hat{j}}{\sqrt{0.5^2 + 1^2}} = -0.45\hat{i} - 0.89\hat{j},\end{aligned}$$

Figure 4: The displacement vector \vec{r}_{P2} .



which we can now put into equation (1) to get \vec{E}_2 :

$$\begin{aligned}
 \vec{E}_2 &= \frac{q_2}{4\pi\epsilon_0 r_{P2}^2} \hat{r}_{P2} \\
 &= \frac{+1nC}{4\pi\epsilon_0 (1.12)^2} (-0.45\hat{i} - 0.89\hat{j}) \\
 &= (-3.22\hat{i} - 6.38\hat{j})V/m.
 \end{aligned} \tag{3}$$

Therefore we can find \vec{E}_{total} :

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 = (-6.38\hat{i} + 3.22\hat{j})V/m + (-3.22\hat{i} - 6.38\hat{j})V/m = (-9.60\hat{i} - 3.16\hat{j})V/m,$$

and from this we can find $|\vec{E}_{total}|$:

$$|\vec{E}_{total}| = \sqrt{9.60^2 + 3.16^2} = 10.11V/m,$$

which is again very close to the value of $10.6V/m$ given by the computer in Figure 2.

Always remember, you must first compute the \vec{E}_1 and \vec{E}_2 *vector* (NOT the magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$) and add them vectorially to get \vec{E}_{total} before using this total vector to find the magnitude $|\vec{E}_{total}|$. NEVER never do the following:

$$|E_{total}| = |\vec{E}_1| + |\vec{E}_2| \quad (\text{WRONG!})$$

2.2 Electric Force

If a charge of $q_3 = -2nC$ is placed at point P (the orange dot), what is the force *vector* on the charge?

Note that the question asked you for the vector, so don't give a scalar as an answer! All you need to do is to use the equation:

$$\begin{aligned}\vec{F}_3 &= q_3 \vec{E}_P \\ &= (-2nC)(-9.60\hat{i} - 3.16\hat{j}) \\ &= (19.2\hat{i} + 6.32\hat{j})nN\end{aligned}$$

As you can see, the signs are flipped because the charge q_3 is negative.

If you are asked to find the *magnitude*, all you have to do is find the magnitude of the force vector you just computed:

$$|\vec{F}_3| = \sqrt{19.2^2 + 6.32^2} = 20.21nN$$